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Optimal Long Term Labor Contracts When
Workers Have Heterogenous Opportunities

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Abstract

This paper considers a two period labor contract. In the first period the worker-firm attachment is made. In the second period the firm's value product is subject to a random shock and the worker obtains a random draw from a distribution over opportunity wages. The firm shock is assumed to be observable by both the firm and its workers. The worker's second period opportunity wage is taken to be privately held information. In this framework the form of the optimal contract written in the first period is analyzed.

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I. Introduction

Much of the recent literature on implicit contracts has stressed the importance of the secondary spot market for labor services in determining the form of the optimal labor contract. The initial papers on implicit contracts by Azariadas (1975), Baily (1974), and Gordon (1974) were all based on the assumption of immobile labor, at the time when the firm specific shock is realized. On the other extreme, these same papers assumed perfect labor mobility, prior to the attachment of workers to their eventual employers. Akerlof and Miyazaki (1980) argue that distinguishing ex ante and ex post labor mobility in this manner is artificial. Holmstrom (1983), in a multi-period model with perfectly mobile labor in each period, demonstrates that long term implicit contracts can still dominate shorter term arrangements. In spite of opportunities for workers elsewhere, a firm can effectively bind workers to the firm by "front end loading" the insurance premiums embodied in the implicit contract. That is, by lowering wage payments during early (and less risky) periods of employment for the promise of higher as well as less variable wages during later (and more risky) periods of employment, the firm provides incentives for workers to remain with the firm. However, the secondary spot market still provides viable alternatives for worker employment. As a result, Holmstrom concludes that contract wages are downward rigid, i.e., they are bounded below by the wage paid on the secondary spot market. This downward rigidity in the wage solves the problem of worker quits.

This paper attempts to extend Holmstrom's analysis by recognizing that the secondary spot market is in fact a collection of markets where a variety of different labor services are traded. Workers who are homogenous with regard to their productivity at their current place of employment may nevertheless command different wages on the secondary spot market because their productivity at alternate places of employment need not be the same. These differences need to be taken into account in optimal contract design. In particular, employment, wages, and severance pay should all be made contingent on the worker's opportunities in the secondary spot market.

Most of the papers that have incorporated some degree of ex post labor mobility with worker heterogeneity, in regard to opportunities external to the firm, have focused on the role of search in determining the form of the implicit contract. When search is costly, it is clear that implicit contracts will not provide for perfect income insurance, since such insurance mitigates the incentives to search. This is a welcome result since, in reality, such perfect insurance is not present. For that reason, the papers of Azariadas, Baily, Gordon, and Holmstrom all assume zero severance payments to laid off workers. But within the internal logic of these models, such an assumption is arbitrary since moral hazard associated with search is not present.

When workers face different costs of search, it is efficient for only some workers to search, i.e., those with low search costs. Hence, when a firm lays off some of its workers, it should not do so indiscriminantly. But it is reasonable to assume that firms cannot observe worker search costs. Consequently an optimal implicit contract must provide the correct incentives to induce only low search cost workers to search. The provision of such incentives reduces the ability of the implicit contract to provide income insurance and also has a negative impact on allocative efficiency. This is the theme of the paper by Geanakoplos and Ito (1984), which focuses on the use of recalls as a device to reveal search costs, and the paper by Arnott, Hosios, and Stiglitz (1983), which focuses on the issue of work sharing versus lay off as an information revealing device.

This paper abstracts from concerns over work sharing and also ignores problems associated with moral hazard related to search. Attention is fixed exclusively on the adverse selection problems which occur in the design of the optimal contract, when workers have different opportunities on the ex post spot market. Kahn (1985) also analyzes the optimal contract design when adverse selection is a problem. Kahn's formulation is similar to the one in this paper, but Kahn views the contract as a deterministic function of worker opportunities. The analysis in this paper provides for the possibility of a much larger spectrum of information revealing devices than have been previously considered, because the contract is viewed as a random function of worker opportunities, i.e., each worker obtains a lottery over employment and layoff with a wage payment associated with the employment outcome and a severance payment associated with the layoff outcome. These lotteries are then made contingent on the workers' opportunities.

The remainder of the paper is organized as follows. Section II sets up the model. Section III provides a solution to the model under the assumption of perfect information ex post. Sections IV, V, and VI analyze the model when the firm remains uninformed about its workers' opportunities ex post. Section VII offers a brief conclusion.

II. Set-up of the Model

To reduce matters to their barest essentials, the model is assumed to last two periods. In the first period each worker forms a contractual attachment with a particular firm. The contract specifies a wage rate in the first period, the probability of being employed in the second period, the wage rate in the second period in the event that the worker is employed, and the severance payment in the second period in the event that the worker is laid off. The latter three parameters are contingent on the firm's second period value product and the worker's second period opportunity wage. This opportunity wage is the price the worker could obtain for his labor services on the second period spot market. The crux of this paper is that workers are heterogenous with regard to their opportunity wages. Further, a particular worker's opportunity wage is privately held information which cannot be observed by the firm.

In the first period, the worker's second period opportunity wage and the firm's second period value product are random variables. Traditional theory suggests that the worker's second period remuneration will be random as well. The worker is assumed to be unable to hedge against this income risk and is therefore taken to be risk averse. The firm is assumed to be risk neutral. Consequently, the long term contract serves, in part, as an insurance contract where workers are guarded against fluctuations in their second period income.[1] In providing this income insurance the firm has several advantages over potential third party insurers. For instance, the firm can more accurately assess the ex ante income risks involved since these risks are determined to a great extent by the terms of the contract. Nevertheless, the firm will not be perfectly informed. As a result of adverse selection problems, the optimal contract does not involve perfect insurance.

There are two types of adverse selection problems discussed in the paper. First, in states where the firm finds it advantageous to lay off some of its workers, i.e., these workers have opportunity wages in excess of their value product at the firm, the severance payment to laid off workers should vary inversely with their opportunity wage. This is in keeping with insuring workers against income risk. In this case, high opportunity wage workers have incentive to misrepresent their opportunity wage in order to secure a higher severance payment. This type of adverse selection is termed layoff adverse selection. Second, it is assumed that workers are free to quit the firm.[2] Hence, it is necessary that the second period wage payment be at least as large as the worker's opportunity wage, if the firm is to retain the worker. In states where the firm wishes to retain its workers, it may offer higher wages to workers with higher opportunity wages, in order to secure their employment. In this case low opportunity wage workers have incentive to misrepresent their opportunity wage in order to be paid more by their present employer. This type of adverse selection is termed quit adverse selection.

The optimal contract deals with layoff adverse selection by punishing workers for lying about their opportunity wage. In this case the probability of employment varies directly with the severance payment and inversely with the worker's opportunity wage. A high opportunity wage worker who attempts to get a high severance payment finds that his chances of being retained are greater. Since his income when he is retained is less than it would be were he separated from the firm, this worker is discouraged from misrepresenting his opportunity wage. The solution to quit adverse selection is similar. A low opportunity wage worker who attempts to get a higher wage finds that his chances of being retained are less. This, too, discourages misrepresentation of the opportunity wage.

For simplicity assume that labor is the firm's only input in production and that the firm operates under stochastic, constant returns to scale. Let k_0 denote the firm's certain (marginal=average) value product in the first period, and k denote the firm's random value product in the second period.

All workers are assumed to be homogenous initially. Each worker is endowed with an indivisible unit of labor service. Hence a worker is employed by at most one firm in any period. In the second period each worker possesses human capital specific to his first period employer. All workers employed by a particular firm in the first period are assumed to possess the same average value product, k, with that firm in the second period.

In the second period the firm's input consists of workers retained from the first period and/or workers hired on the second period spot market. Workers on the spot market have either been laid off by their previous employer or have quit their previous job.[3] Such workers are assumed to be heterogenous in regard to the complementary attribute-skill vector that they bring to their new job. For simplicity assume that each firm requires workers with a particular attribute-skll vector. Though the workers on the spot market who fulfill these attribute-skill requirements are different from the retained workers who possess firm specific human capital, it is convenient to assume that these workers are perfect substitutes as inputs in second period production. Hence, it is assumed that the average value product of any worker hired on the spot market is αk, where α is a positive scalar. (α may be a random variable.) Since the second period spot market is assumed to be competitive and production is assumed to occur under constant returns to scale, the wage rate for workers with this particular attribute skill vector must be at least αk. Because workers hired on the second period spot market do not alter the firm's profits, it is convenient to ignore them in the firm's decision problem.

At the start of the second period each worker obtains a random draw from a distribution over attribute-skill vectors. Since the attribute-skill requirements of firms are assumed to differ and the value products of firms differ as well, workers with different attribute-skill vectors will have different opportunity wages. The opportunity wage of a given worker is the maximum average value product among those firms which require the attribute-skill vector that the given worker possesses. Since the focus of this paper is the optimal labor contract of a particular firm, it is simply assumed that each worker obtains a random draw from a distribution over opportunity wages. Let w^+ denote the opportunity wage of a representative worker. Let F denote the joint distribution function over workers' opportunity wages and the firm's average value product.

A long-term labor contract between the firm and a given worker is a first period wage, w_0, and a function, (e,w,s), where $e(w^+,k)$ is the probability of being retained in the second period, $w(w^+,k)$ is the wage payment if retained, and $s(w^+,k)$ is the severance payment if laid off, when w^+ is the worker's opportunity wage and k is the firm's average value product. For a long term contract to attract the worker initially, it must grant expected utility at least as large as the expected utility the worker can obtain elsewhere. Let u be the worker's one period utility function. It is assumed that $u \varepsilon C^2$, $u' > 0$, and $u'' < 0$. Let V be the parametrically given, two period expected utility level that the worker can obtain elsewhere. Then the expected utility constraint on the long term contract is given by

$$(1) \quad u(w_0) + \int [e(w^+,k)u(w(w^+,k)) + (1-e(w^+,k))u(w^+ + s(w^+,k))]dF(w^+,k) \geq V.$$

Since workers are free to quit, it is also required that a feasible contract satisfy

(2) $w(w^+,k) \geq w^+$ for all w^+,k.

Since workers separated from their original employer are under no obligation to that employer, it is required that

(3) $s(w^+,k) \geq 0$ for all w^+,k.

Since $e(w^+,k)$ is the probability of being retained, it is required that

(4) $0 \leq e(w^+,k) \leq 1$ for all w^+,k.

It is assumed that k is observable by both the firm and the workers it hired in the first period.[4] If w^+ is also symmetrically observed, then the problem for the firm maximizing expected profit can be written as

(5) maximize $k_0 - w_0 + \int [e(w^+,k)[k-w(w^+,k)] - (1-e(w^+,k))s(w^+,k)]dF(w^+,k)$

subject to (1), (2), (3), and (4).

Note: The above problem is written as if the firm hires only one worker in the first period. Since each worker possesses the same utility function, is risk averse, and faces the same period one probability distribution over the opportunity wages in the second period, it is optimal for the firm to offer all workers the same contract in the first period. Furthermore, the firm must make zero expected profits since it operates under stochastic, constant returns to scale. In equilibrium, the parameter V adjusts so that this condition is satisfied. Hence the scale of the firm is indeterminante.

In order to understand the problem when the firm does not observe w^+, the model is first solved under the assumption that workers and the firm have symmetric information. The solution to (5) is characterized in the next section.

III. Symmetric Information

The problem given in (5) is solved in two stages. First, the second period problem is solved conditional on the firm's average value product being k and assuming that the firm has promised the worker expected utility equal to $\bar{U}(k)$ in this event. Then the optimal $\bar{U}(k)$ is found. The second period problem is given by

$$(6)\quad \text{maximize} \int [e(w^+)[k-w(w^+)] - (1-e(w^+))s(w^+)]dF(w^+|k)$$

$$\text{subject to} \int [e(w^+)u(w(w^+)) + (1-e(w^+))u(w^++s(w^+))]dF(w^+|k) \geq \bar{U}(k),$$

$$w(w^+) \geq w^+,\ s(w^+) \geq 0, \text{ and } 0 \leq e(w^+) \leq 1.$$

Note: The explicit dependence of contract variables on k has been dropped to enhance readability.

Assume that F is continuously differentiable and let the conditional density be denoted by $f(w^+|k)$. Let the support of this density be the interval $[\underline{w},\bar{w}]$. Define the state variable $U(w^+)$ by

$$(7)\quad U(w^+) = \int_{\underline{w}}^{w^+} [e(\tilde{w})u(w(\tilde{w})) + (1-e(\tilde{w}))u(\tilde{w}+s(\tilde{w}))]f(\tilde{w}|k)d\tilde{w}.$$

Letting the control variables be $e(w^+)$, $w(w^+)$, and $s(w^+)$, the Hamiltonian for the problem is

$$(8) \quad H = \{e(w^+)[k-w(w^+) + \lambda u(w(w^+))] + (1-e(w^+))[-s(w^+) + \lambda u(w^+ + s(w^+))]\}f(w^+|k)$$

Since the Hamiltonian does not depend on the state variable, the costate variable, λ, is a constant. The remaining necessary conditions are

$$(9) \qquad e(w^+) = 0 \quad \text{if } k-w(w^+) + \lambda u(w(w^+)) < -s(w^+) + \lambda u(w^+ + s(w^+)),$$

$$e(w^+) = 1 \quad \text{if } k-w(w^+) + \lambda u(w(w^+)) > -s(w^+) + \lambda u(w^+ + s(w^+)),$$

and $\quad 0 \leq e(w^+) \leq 1.$

$$(10) \qquad e(w^+)[-1 + \lambda u'(w(w^+))] \leq 0, \; w(w^+) \geq w^+, \text{ and}$$

$$(w(w^+)-w^+)e(w^+)[-1 + \lambda u'(w(w^+))] = 0.$$

$$(11) \qquad (1-e(w^+))[-1 + \lambda u'(w^+ + s(w^+))] \leq 0, \; s(w^+) \geq 0, \text{ and}$$

$$s(w^+)(1-e(w^+))[-1 + \lambda u'(w^+ + s(w^+ + s(w^+))] = 0.$$

$$(12) \qquad U(\overline{w}) \geq \overline{U}(k).$$

<u>Theorem 1</u>: The solution to (6) takes the following form

$$(13) \qquad e(w^+) = 1 \quad \text{for } w^+ < k,$$

$$e(w^+) = 0 \quad \text{for } w^+ > k.$$

$$(14) \qquad w(w^+) = \max(w_c, w^+) \text{ when } e(w^+) > 0.$$

(15) $\quad s(w^+) = \max(w_c - w^+, 0)$ when $e(w^+) < 1$.

Proof: The constraint (12) is binding as long as $\overline{U}(k) > \int_{\underline{w}}^{\overline{w}} u(w^+)f(w^+|k)dw^+$. When this inequality holds $\lambda > 0$. Assuming this, let w_c be implicitly defined by $u'(w_c) = \frac{1}{\lambda}$. Let $G \equiv k - w + \lambda u(w)$. G is strictly concave in w. It follows that $\underset{w \geq w^+}{\operatorname{argmax}}\, G = \max(w_c, w^+)$. Let $L \equiv -s + \lambda u(w^+ + s)$. L is strictly concave in s. It follows that $\underset{s \geq 0}{\operatorname{argmax}}\, L = \max(w_c - w^+, 0)$. Furthermore, $\max_{w \geq w^+} G \gtreqless \max_{s \geq 0} L$ as $k \gtreqless w^+$. Since the controls are chosen to maximize the Hamiltonian, H, the theorem follows directly from the above.

Theorem 1 says that the optimal contract satisfies a productive efficiency condition, (13), and an insurance condition, (14) and (15). That is, workers are retained by the firm if their marginal value product is greater than their opportunity wage and workers are laid off if their marginal value product is less than their opportunity wage. Furthermore, a worker's income is independent of his opportunity wage as long as the opportunity wage is below a certain level. Note that there is a downward jump discontinuity in the optimal control, $e(w^+)$, at $w^+ = k$.

We proceed to a determination of the optimal $\overline{U}(k)$. Let $\pi(k, \overline{U}(k))$ be the value of the objective in (6) when evaluated along the optimal program. The optimal contract problem given in (5) can be rewritten as

$$(16)\qquad \text{maximize } k_0 - w_0 + \int_{\underline{k}}^{\overline{k}} \pi(k,\overline{U}(k)) f_K(k)dk$$

$$\text{subject to } u(w_0) + \int_{\underline{k}}^{\overline{k}} \overline{U}(k) f_K(k)dk \geq V,$$

where $f_K(k)$ is the marginal density of k and $[\underline{k},\overline{k}]$ is the support of this density. This problem can be viewed as a control problem with control $\overline{U}(k)$ and state variable $X(k)$, where

$$(17)\qquad X(k) = \int_{\underline{k}}^{k} \overline{U}(h) f_K(h)dh.$$

The Hamiltonian for this problem is

$$(18)\qquad H = [\pi(k,\overline{U}(k)) + \mu\overline{U}(k)] f_K(k).$$

The control $\overline{U}(k)$ is constrained by

$$(19)\qquad \overline{U}(k) \geq \int_{\underline{w}}^{\overline{w}} U(w^+) f(w^+|k)dw^+.$$

Since the state variable $X(k)$ does not enter into the Hamiltonian, the costate variable μ is a constant. The remaining necessary condition is.

$$(20)\qquad \pi_{\overline{U}}(k,\overline{U}(k)) + \mu \leq 0$$

with equality when (19) holds as a strict inequality.

As long as (19) is satisfied, $\pi_{\overline{U}}(k,\overline{U}(k)) = -\lambda(k)$, where $\lambda(k)$ is the costate variable of the Hamiltonian given by (8). Furthermore, when $\overline{U} = \int_{\underline{w}}^{\overline{w}} u(w^+)f(w^+|k)dw^+$, $\pi_{\overline{U}}(k,\overline{U}) + \mu > 0$ as long as $u'(\underline{w}(k)) > \frac{1}{\mu}$. Finally, the optimal first period wage, w_0, satisfies

$$(21) \qquad u'(w_0) = \frac{1}{\mu}.$$

This condition, along with Theorem 1, implies the following result.

Theorem 2: The solution to (5) takes the following form.

$$(21) \qquad e(w^+,k) = 1 \quad \text{for } w^+ < k,$$

$$e(w^+,k) = 0 \quad \text{for } w^+ > k.$$

$$(22) \qquad w(w^+,k) = \max(w_0,w^+) \text{ when } e(w^+,k) > 0.$$

$$(23) \qquad s(w^+,k) = \max(w_0-w^+,0) \text{ when } e(w^+,k) < 1.$$

IV. Privately Observed Opportunity Wages

When w^+ is not observed by the firm the contract specified in the previous section may not be feasible. From the literature on incentive compatibility,[5] an optimal contract can be found by imposing the self-selection constraints

$$(24) \qquad e(w^+)u(w(w^+)) + (1-e(w^+))u(w^++s(w^+)) \geq$$

$$e(\tilde{w})u(w(\tilde{w})) + (1-e(\tilde{w}))u(w^++s(\tilde{w})) \text{ for all } w^+,\tilde{w} \in [\underline{w},\overline{w}].$$

Viewing the right hand side of (24) as a function of $\tilde{w}$, the above constraints require that this function is maximized on $[\underline{w},\overline{w}]$ at w^+. The first order necessary condition for such a maximum is given by

$$(25)\quad \dot{e}(w^+)[u(w(w^+))-u(w^++s(w^+))]+\dot{w}(w^+)e(w^+)u'(w(w^+))+\dot{s}(w^+)(1-e(w^+))u'(w^++s(w^+))=$$

where a dot above a variable indicates a derivative with respect to w^+. For (24) to hold, (25) must hold for all $w^+ \varepsilon [\underline{w},\overline{w}]$. Since it is more natural to consider e, w, and s as controls (25) will not be imposed directly. Instead a new state variable, $Y(w^+)$, is introduced, where $\dot{Y}(w^+) = (1-e(w^+))u'(w^++s(w^+))$. Then the following state-control constraint is imposed.

$$(25')\qquad Y(w^+) = e(w^+)u(w(w^+)) + (1-e(w^+))u(w^++s(w^+)).$$

Differentiation of (25') with respect to w^+ yields (25) after a substitution for $\dot{Y}(w^+)$. In what follows, the problem specified in (6) is analyzed when (25') is treated as an additional constraint.

Before proceeding further, it is appropriate at this juncture to discuss sufficiency. That is, under what conditions does a contract which satisfies (25) also satisfy (24)? A natural sufficiency condition is to require that the right hand side of (24) be pseudo-concave in $\tilde{w}$ (single-peaked) with peak at w^+. Formally, this condition is given by

$$(26)\quad \dot{e}(\tilde{w})[u(w(\tilde{w}))-u(w^++s(\tilde{w}))] + \dot{w}(\tilde{w})e(\tilde{w})u'(w(\tilde{w})) + \dot{s}(\tilde{w})(1-e(\tilde{w}))u'(w^++s(\tilde{w}))$$

$$\gtreqless 0 \text{ as } w^+ \gtreqless \tilde{w}.$$

Since (25) must hold at $\tilde{w}$, (26) is equivalent to

$$(26') \quad \dot{e}(\tilde{w})[u(\tilde{w}+s(\tilde{w})) - u(w^{+}+s(\tilde{w}))]$$

$$+ \dot{s}(\tilde{w})(1-e(\tilde{w}))[u'(w^{+}+s(\tilde{w}))-u'(\tilde{w}+s(\tilde{w}))] \gtreqless 0 \text{ as } w^{+} \gtreqless \tilde{w}.$$

Note that (26') is satisfied if $\dot{e}(\tilde{w})$, $\dot{s}(\tilde{w}) \leq 0$ for all $\tilde{w} \in [\underline{w},\overline{w}]$. Contracts which satisfy this latter property will be called montonic contracts. The results which follow provide sufficient conditions under which the optimal contract is a monotonic contract.[6]

The firm's second period problem when it cannot observe the worker's opportunity wage can be written as

$$(27) \quad \underset{\substack{0 \leq e \leq 1 \\ w \geq w^{+},\ s \geq 0}}{\text{maximize}} \int_{\underline{w}}^{\overline{w}} \{ke-[we+s(1-e)]\}f(w^{+}|k)dw^{+}$$

subject to

$$\dot{U} = Yf(w^{+}|k),\ U(\underline{w}) = 0,\ U(\overline{w}) = \overline{U}(k)$$

$$\dot{Y} = (1-e)u'(w^{+}+s),\ Y = eu(w) + (1-e)u(w^{+}+s),\ Y(\overline{w}) \geq u(\overline{w}).$$

Note: In the above the explicit dependence of controls and state variables on w^{+} has been dropped.

The Hamiltonian for this problem is

$$(28) \quad H = \{ke-[we+s(1-e)]\}f(w^{+}|k) + \lambda_{u}Yf(w^{+}|k) + \lambda_{y}(1-e)u'(w^{+}+s)$$

Since (25') must be satisfied the associated Lagrangian is

$$(29) \quad L = H + \mu[eu(w)+(1-e)u(w^{+}+s) - Y].$$

Since the state variable U does not enter in L, the costate variable, λ_u, is a constant. The costate equation for λ_y is given by

(30) $\quad -\dot{\lambda}_y = \lambda_u f(w^+|k) - \mu.$

The remaining necessary conditions are

(31) $\quad e = 0$ if $[k-w+s]f(w^+|k) - \lambda_y u'(w^+ +s) + \mu[u(w)-u(w^+ +s)] < 0$

$\quad e = 1$ if $[k-w+s]f(w^+|k) - \lambda_y u'(u^+ +s) + \mu[u(w)-u(w^+ +s)] > 0$

and $\quad 0 \leq e \leq 1.$

(32) $\quad e[-f(w^+|k) + \mu u'(w)] \leq 0,\ w \geq w^+$ and

$\quad (w-w^+)e[-f(w^+|k) + \mu u'(w)] = 0.$

(33) $\quad (1-e)[-f(w^+|k) + \lambda_y u''(w^+ +s) + \mu u'(w^+ +s)] \leq 0,\ s \geq 0,$ and

$\quad s(1-e)[-f(w^+|k) + \lambda_y u''(w^+ +s) + \mu u'(w^+ +s)] = 0.$

There is also a transversality condition.

(34) $\quad \lambda_y(\underline{w}) = 0,\ Y(\overline{w}) \geq u(\overline{w}),\ \lambda_y(\overline{w})[Y(\overline{w})-u(\overline{w})] = 0.$

The form of the solution depends on the relationship between the parameters $\underline{w}$, $\overline{w}$, k, and $u^{-1}(\overline{U}(k))$. Attention will be restricted to the case $\underline{w} < k$ so that the firm will find it efficient to always retain some of its workers.

If the solution to the symmetric information case happens to satisfy (24), then naturally this continues to be the solution when w^+ is privately observed. When $\overline{w} \leq k$ and $\overline{w} \leq u^{-1}(\overline{U}(k))$ a full employment, constant wage contract is optimal. When $\bar{w} > k$ let

$$\int_{\underline{w}}^{k} u(k)f(w^+|k)dw^+ + \int_{k}^{\bar{w}} u(w^+)f(w^+|k)dw^+ = U^{BE}.$$

When $\bar{U}(k) = U^{BE}$, the optimal contract is one which fully employs all workers whose opportunity wage is less than k at a wage equal to k and lays off without severance pay all workers with a higher opportunity wage.

In what follows it will be convenient to distinguish two cases. Though it is common to call firm induced separations layoffs and worker induced separations quits, these terms will take on a slightly different meaning in this paper. The optimal contract will be characterized by layoffs if $\bar{U}(k) > U^{BE}$. The optimal contract will be characterized by quits if $\bar{U}(k) < U^{BE}$ or if $u^{-1}(\bar{U}(k)) < \bar{w} \leq k$.

V. Layoffs

Any feasible contract has the property that if $Y(w^+) = u(w^+)$, then $Y(\tilde{w}) = u(\tilde{w})$ and $e(\tilde{w}) = s(\tilde{w}) = 0$, for $w^+ < \tilde{w} \leq \bar{w}$. This follows since $Y(w^+) = eu(w) + (1-e)u(w^+ + s) \geq u(w^+)$ and $\dot{Y}(w^+) = (1-e)u'(w^+ + s) \leq (1-e)u'(w^+) \leq u'(w^+)$. Since $\underline{w} < k$ by assumption, an optimal contract satisfies $Y(\underline{w}) > u(\underline{w}^+)$.

The following lemma shows that, with layoffs, all workers with opportunity wage less than k are fully employed. In fact, workers whose opportunity wage is near to but above k are also fully employed.

<u>Lemma 1</u>: Let $\bar{U}(k) > U^{BE}$. Then there exists $\varepsilon > 0$ such that, in the optimal contract, $e(w^+) = 1$ for $w^+ \leq k + \varepsilon$. Furthermore, $\dot{\lambda}_y < 0$ for $w^+ \leq k + \varepsilon$.

Proof: We begin by noting that $e(\underline{w}) = 1$. To see this let $h(s) = [k-w+s] + \frac{u(w) - u(w^+ +s)}{u'(w)}$. h is convex and is minimized at $s = w - w^+$. $h(w-w^+) = k-w^+$. When $w^+ < k$, h is postive. As long as $e > 0$ and $w > w^+$, $\mu = \frac{f(w^+|k)}{u'(w)}$. In this case (31) requires that $e = 1$ if $h(s)f(w^+|k) - \lambda_y u'(w^+ +s) > 0$. This inequality clearly holds when $w^+ < k$ and $\lambda_y \leq 0$. In particular, the inequality holds at $\underline{w}$ since $\lambda_y(\underline{w}) = 0$, by (34). Since λ_y is continuous as long as $Y(w^+) > u(w^+)$, the inequality must hold over some interval bounded below by $\underline{w}$.

On an interval where $e = 1$, $\dot{Y} = 0$ and consequently $\dot{w} = 0$. Substituting (32) into (30) yields $\dot{\lambda}_y(\underline{w}) = [\frac{1}{u'(w(\underline{w}))} - \lambda_u]f(\underline{w}|k)$. Furthermore, if $w(w^+) = w(\underline{w}) > w^+$ then $\dot{\lambda}_y(w^+) = \frac{f(w^+|k)}{f(\underline{w}|k)} \dot{\lambda}_y(\underline{w})$. It follows that if $\dot{\lambda}_y(\underline{w}) \leq 0$, then $e(w^+) = 1$ and $w(w^+) = w(\underline{w})$ for $w^+ < \min[k,w(\underline{w})]$. By the first paragraph of this section, $\dot{\lambda}_y(\underline{w}) \leq 0$ implies $k < w(\underline{w})$, since $\overline{U}(k) > U^{BE}$.

We proceed to show that $\dot{\lambda}_y(\underline{w}) > 0$ is not possible when $\overline{U}(k) > U^{BE}$. Suppose on the contrary that $\dot{\lambda}_y(\underline{w}) > 0$. If $\lambda_y > 0$ and $0 < e < 1$, then $w \geq w^+ + s$, by (32) and (33). Hence $w \geq w(\underline{w})$ when $\lambda_y > 0$ and $e > 0$, since Y is nondecreasing. In this case $\frac{1}{u'(w)} \geq \frac{1}{u'(w(\underline{w}))}$. Hence $\dot{\lambda}_y(w^+) \geq \dot{\lambda}_y(\underline{w}) \frac{f(w^+|k)}{f(\underline{w}|k)} > 0$. If $e = 0$ and $s > 0$, (30) and (33) yield:

$$\dot{\lambda}_y u'(w^+ +s) + \lambda_y u''(w^+ +s) = [1-\lambda_u u'(w^+ +s)]f(w^+|k).$$

Since $w^+ +s \geq w(\underline{w})$ the right hand side of the above equality is positive if $\dot{\lambda}_y(\underline{w}) > 0$. Hence, as long as $\lambda_y \geq 0$, $\dot{\lambda}_y > 0$ in this case. Consequently $\lambda_y(w^+), \dot{\lambda}_y(w^+) > 0$ for $w^+ > \underline{w}$, when $\dot{\lambda}_y(\underline{w}) > 0$ and $Y(w^+) > u(w^+)$.

It follows that when $\dot{\lambda}_y(\underline{w}) > 0$ then $Y(\overline{w}) = u(\overline{w})$. Otherwise, $\lambda_y(\overline{w}) > 0$ by the previous paragraph. But this violates the transversality condition (34). Clearly, $\dot{\lambda}_y(\underline{w}) \leq 0$ when $\overline{U}(k) > u(\overline{w})$.

When $U^{BE} < \overline{U}(k) \leq u(\overline{w})$ a different argument is needed to show why $\dot{\lambda}_y(\underline{w}) > 0$ is not optimal. Note that if $\dot{\lambda}_y(\underline{w}) > 0$, then $e(w^+) < 1$ for w^+ near to but less than k. This follows since $Y(k) > u(k)$, so $\dot{\lambda}_y(\underline{w}) > 0$ implies $\lambda_y(k) > 0$. Hence, if $e(k) = 1$ the Lagrangian could be increased by instead setting $e(k) = 0$ and $s(k) = w(k)-k$. In other words, if $\dot{\lambda}_y(\underline{w}) > 0$ then some workers with opportunity wage less than k are separated from the firm. In this case we construct an alternate contract, where all of these workers are retained, such that the firm makes greater profits.

Let (e^a,w^a,s^a) denote this alternate contract. Let δ be such that $0 \leq \delta \leq Y(k)-u(k)$. Let $Y^*(k) = Y(k)-\delta$ and implicitly define $Y^*(w^+)$ for $w^+ > k$ by:

$$Y^*(w^+) = Y^*(k) + \int_k^{w^+} (1-e^a(\tilde{w}))u'(\tilde{w}+s^a(\tilde{w}))d\tilde{w}.$$

If $e(w^+) > 0$, let $w^*(w^+) = u^{-1}(u(w) + \frac{[Y^*(w^+)-Y(w^+)]}{e})$. If $w^*(w^+) \geq w^+ + s(w^+)$, then let $e^a(w^+) = e(w^+)$, $w^a(w^+) = w^*(w^+)$, and $s^a(w^+) = s(w^+)$. If $w^*(w^+) < w^+ + s(w^+)$ but $Y^*(w^+) > u(w^+)$, then let $e^a(w^+) = e(w^+)$ and $w^a(w^+) = w^+ + s^a(w^+) = u^{-1}(Y^*(w^+))$. If $Y^*(w^+) = u(w^+)$ let $e^a(w^+) = 0$ and $s^a(w^+) = 0$. With this construction $0 \leq Y(w^+) - Y^*(w^+) \leq \delta$. Furthermore $(e^a(w^+),w^a(w^+),s^a(w^+)) \leq (e(w^+),w(w^+),s(w^+))$.

For $w^+ < k$, let $e^a(w^+) = 1$ and

$$w^a(w^+) = u^{-1}([\bar{U}(k) - \int_k^{\bar{w}} Y^*(\tilde{w})f(\tilde{w}|k)d\tilde{w}]/\int_{\underline{w}}^k f(\tilde{w}|k)d\tilde{w}).$$

When $\delta = 0$, $Y^*(k) = Y(k)$ while $u(w^a(w^+)) = \int_{\underline{w}}^k Y(\tilde{w})f(\tilde{w}|k)d\tilde{w}/\int_{\underline{w}}^k f(\tilde{w}|k)d\tilde{w} < Y(k)$, for $w^+ < k$. When $\delta = Y(k)-u(k)$, $Y^*(k) = u(k)$, while $u(w^a(w^+)) = [\bar{U}(k)-\int_k^{\bar{w}} u(\tilde{w})f(\tilde{w}|k)d\tilde{w}]/\int_{\underline{w}}^k f(\tilde{w}|k)d\tilde{w} > u(k)$, for $w^+ < k$. It follows that for some intermediate value of δ; $Y^*(k) = u(w^a(w^+))$, for $w^+ < k$. Let δ^* denote this intermediate value. The alternate contract determined by δ^* yields a feasible contract.

When $\delta = 0$ the alternate contract is more profitable than the original contract. This follows since the contracts coincide, for $w^+ \geq k$, while the alternate contract satisfies the conditions of Theorem 1, for $w^+ < k$. Furthermore, the profitability of the alternate contract is increasing in δ, for $\delta \leq \delta^*$. This follows by a straightforward income smoothing argument.[7] Hence, the alternate contract determined by δ^* is more profitable than the original contract. It follows that $\dot{\lambda}_y(\underline{w}) > 0$ is not possible when $\bar{U}(k) > U^{BE}$.

The above argument can be extended to show that $e(w^+) = 1$ for $w^+ < k + \varepsilon$, for some $\varepsilon > 0$.[8] It follows that $\lambda_y(w^+) < 0$ for $k < w^+ < k + \varepsilon$, since (31) must hold. But this is not possible if $\dot{\lambda}_y(\underline{w}) = 0$. Hence $\dot{\lambda}_y(\underline{w}) < 0$ and $\dot{\lambda}_y(w^+) < 0$ for $w^+ \leq k + \varepsilon$.

To summarize the above, when there are layoffs there is actually overemployment, measured from the symmetric information solution. Employment is utilized as an incentive compatible vehicle for income smoothing.

We turn to the form of the optimal contract when $e(w^+) < 1$. The Lagrangian is singular in e and consequently one must be careful about bang-bang solutions. The next lemma provides sufficient conditions for there to be a unique vector of controls which maximize the Lagrangian. Since the Lagrangian must be continuous in w^+, it follows that the controls must be continuous in w^+ as long as the conditions of the lemma are met.

Lemma 2: If u exhibits nondecreasing absolute risk aversion, $Y > u(w^+)$ and $\lambda_y < 0$ then there is a unique vector of controls which maximize the Hamiltonian given by (28) and satisfy the state control constraint:

$$Y = eu(w) + (1-e)u(w^+ + s).$$

Proof: Consider the problem: $\underset{w \geq w^+}{\text{maximize}}\ [k-w]f(w^+|k) + \mu u(w)$. As long as $\mu > 0$, the objective is strictly concave. Consequently there is a unique solution which we denote by $w(\mu)$. $w(\mu)$ is nondecreasing in μ and strictly increasing if $w(\mu) > w^+$. Let $h(\mu) = [k-w(\mu)]f(w^+|k) + \mu u(w(\mu))$. Note that $h'(\mu) = u(w(\mu))$, by the envelope theorem. Now consider the problem: $\underset{s \geq 0}{\text{maximize}}\ -sf(w^+|k) + \lambda_y u'(w^+ + s) + \mu u(w^+ + s)$. Since nondecreasing absolute risk aversion implies $u'''(w^+ + s) \geq 0$, the objective is strictly concave. Consequently there is a unique solution which we denote by $s(\mu)$. It follows from the first order conditions and the assumption that $\lambda_y < 0$ that $w(\mu) \leq w^+ + s(\mu)$, with strict inequality when $s(\mu) > 0$. $s(\mu)$ is nondecreasing in μ and strictly increasing when $s(\mu) > 0$. Let $g(\mu) = -s(\mu)f(w^+|k) + \lambda_y u'(w^+ + s(\mu)) + \mu u(w^+ + s(\mu))$. $g'(\mu) = u(w^+ + s(\mu))$. Note that $g'(\mu) > h'(\mu)$ as long as

$s(\mu) > 0$. From (31), if $e(w^+) = 1$ then $h(\mu(u^+)) \geq g(\mu(w^+))$. In this case $w(\mu(w^+)) = u^{-1}(Y(w^+))$. It follows that for $\mu < \mu(w^+)$, $g(\mu) < h(\mu)$, while for $\mu > u(w^+)$, $w^+ + s(\mu) > w(\mu) > u^{-1}(Y(u^+))$. Hence there is a unique vector of controls in this case. If $e(w^+) = 0$, then $h(\mu(w^+)) \leq g(\mu(w^+))$, again by (31). In this case $w^+ + s(\mu(w^+)) = u^{-1}(Y(w^+))$. It follows that for $\mu < \mu(w^+)$, $w(\mu) < w^+ + s(\mu) < u^{-1}(Y(w^+))$, while for $\mu > \mu(u^+)$, $h(\mu) < g(\mu)$. There is also a unique vector of controls in this case. Finally, if $0 < e(w^+) < 1$ then $h(\mu(w^+)) = g(\mu(w^+))$, by (31), and $w(\mu(w^+)) < u^{-1}(Y(w^+)) < w^+ + s(\mu(w^+))$. It follows that for $\mu < \mu(w^+)$, $h(\mu) > g(\mu)$ but $w(\mu) < u^{-1}(Y(w^+))$, while for $\mu > \mu(w^+)$, $h(\mu) < g(\mu)$ but $u^{-1}(Y(w^+)) < w^+ + s((\mu))$. Hence there is also a unique solution in this case.

When $0 < e < 1$, $w > w^+$, and $s > 0$, (31)-(33) yield

(35) $$\mu = \frac{f(w^+|k)}{u'(w)},$$

(36) $$\lambda_y = \frac{u'(w^+ + s)}{u''(w^+ + s)} \left[\frac{1}{u'(w^+ + s)} - \frac{1}{u'(w)}\right] f(w^+|k), \text{ and}$$

(37) $$k - w + s + \frac{[u(w) - u(w^+ + s)]}{u'(w)} + \frac{u'(w^+ + s)^2}{u''(w^+ + s)} \left[\frac{1}{u'(w)} - \frac{1}{u'(w^+ + s)}\right] = 0$$

It follows that the total derivatives of the right hand side of (36) with respect to w^+ must equal $f(w^+|k)[\frac{1}{u'(w)} - \lambda_u]$, from (30) and (35). It also follows that the total derivative of the left hand side of (37) with respect to w^+ must equal zero. This yields two nonhomogenous equations in $\dot{w}$ and $\dot{s}$. The solution to these equations can then be substituted into (25) to determine $\dot{e}$. Below we proceed to sufficient conditions for $\dot{s}$, $\dot{e} \leq 0$, i.e., for the optimal contract to be a monotonic contract.

Differentiating (36) and substituting into (30) yields

$$(36') \qquad A_w\dot{w} + A_s\dot{s} + A_{w^+} = f(w^+|k)\left[\frac{1}{u'(w)} - \lambda_u\right],$$

where $A_w = \dfrac{u'(w^+ + s)}{u''(w^+ + s)} \dfrac{u''(w)}{u'(w)^2} f(w^+|k)$,

$$A_s = \left\{\frac{-1}{u'(w^+ + s)} + \frac{[u''(w^+ + s)^2 - u'(w^+ + s)u'''(w^+ + s)]}{u''(w^+ + s)^2}\left[\frac{1}{u'(w^+ + s)} - \frac{1}{u'(w)}\right]\right\} f(w^+|k),$$

and $A_{w^+} = A_s + \dfrac{u'(w^+ + s)}{u''(w^+ + s)}\left[\dfrac{1}{u'(w^+ + s)} - \dfrac{1}{u'(w)}\right] f'(w^+|k)$.

Likewise, differentiating (37) yields

$$(37') \qquad B_w\dot{w} + B_s\dot{s} + B_{w^+} = 0,$$

where $B_w = -\dfrac{u''(w)}{u'(w)^2}[u(w) - u(w^+ + s)] - \dfrac{u'(w^+ + s)^2}{u''(w^+ + s)} \dfrac{u''(w)}{u'(w)^2}$,

$$B_s = 1 + \frac{[u''(w^+ + s)^2 u'(w^+ + s) - u'(w^+ + s)^2 u'''(w^+ + s)]}{u'(w^+ + s)^2}\left[\frac{1}{u'(w)} - \frac{1}{u'(w^+ + s)}\right],$$

and $B_{w^+} = B_s - 1$.

Note that $B_s f(w^+|k) = -A_s u'(w^+ + s)$. Also note that $w < w^+ + s$ when $\lambda_y < 0$, from (32) and (33). Hence, when $\lambda_y < 0$ and u exhibits non-increasing absolute risk aversion we have: $A_s < 0$, $\dfrac{-A_w}{A_s} < \dfrac{-B_w}{B_s}$, and $0 > \dfrac{-B_{w^+}}{B_s} > -1$. From this it follows that $\dot{w} \leq 0$, $\dot{s} < 0$ if

$\frac{-A_{w^+} + f(w^+|k)[\frac{1}{u'(w)} - \lambda_u]}{A_s} \leq \frac{-B_{w^+}}{B_s}$.[9] After some manipulation it can be shown that this inequality is equivalent to:

$$(38) \qquad \frac{u'(w^++s)}{u''(w^++s)}\left[\frac{1}{u'(w^++s)} - \frac{1}{u'(w)}\right]f'(w^+|k) \leq \left[\frac{1}{u'(w)} + \frac{1}{u'(w^++s)} - \lambda_u\right]f(w^+|k).$$

When (38) holds $\dot{e}$, $\dot{s} < 0$. When (38) holds with equality $\dot{w} = 0$ and $-1 < \dot{s}$.

Let $w_* = \sup\{w^+ \colon \; e(w^+) = 1 \text{ and } w(w^+) = w(\underline{w})\}$. $w_* < \overline{w}$ since a full employment contract can be readily shown to be dominated by a contract with layoffs when $U^{BE} < \overline{U}(k)$. Clearly, $\dot{e}(w_*) \leq 0$ since $e \leq 1$ is required. From (25) it follows that $\dot{w}(w_*) \leq 0$. Hence, (38) must hold at w_*. Let $w^* = \inf\{w^+ \colon \; e(w^+) = 0\}$. Since lemma 2 rules out jumps in the controls at w^* it follows that $w_* < w^*$. A sufficient condition for (38) to hold on $[w_*, w^*]$ is that the distribution over opportunity wages is uniform.[10] To see this note that in this case the left hand side of (38) is identically zero while the derivative of the right hand side is positive when the right hand side is near zero.

We are now in a position to describe the form of the optimal contract with layoffs. This description is given by the next theorem.

<u>Theorem 3</u>: Suppose $\overline{U}(k) > U^{BE}$ and u exhibits nondecreasing absolute risk aversion. Suppose w_*, w^* are defined as above and the optimal contract satisfies (38) for $w^+ \geq w_*$ as well as $\lim_{w^+ \uparrow w^*} w(w^+) > w^*$.

Then $k < w_* < w^* < \overline{w}$. In addition the optimal contract satisfies:

$e(w^+) = 1$, $w(w^+) = w(\underline{w})$ for $w^+ \leq w_*$;

$w(w^+) < w^+ + s(w^+)$, $\dot{e}(w^+)$ and $\dot{s}(w^+) < 0$, and

$\dot{w}(w^+) \leq 0$ for $w_* < w^+ < w^*$; and

$e(w^+) = 0$, $s(w^+) = s(w^*)$ for $w^+ \geq w^*$.

Proof: $k < w_*$ by Lemma 1 and $w_* < w^*$ by Lemma 2. Let $\hat{w} = \inf\{w^+ > w_*: \lambda_y(w^+) \geq 0\}$. We know from Lemma 1 that $\lambda_y(w_*) < 0$. We show that $\hat{w} > w^*$.

Suppose instead that $\hat{w} \leq w^*$. If $w(w^+) > w^+$ for $w_* < w^+ < \hat{w}$, then $\dot{w}(w^+) \leq 0$ as well, since (38) is satisfied. Hence $w(w^+) \leq w(w_*)$ and

$$\dot{\lambda}_y(w^+) = [\frac{1}{u'(w(w^+))} - \lambda_u]f(w^+|k) \leq [\frac{1}{u'(w(w_*))} - \lambda_u]f(w^+|k) \leq$$

$\dot{\lambda}_y(w_*) \dfrac{f(w^+|k)}{f(w_*|k)} < 0$. So $\lambda_y(\hat{w}) < 0$, which is a contradiction. On the other hand, it is not possible that $w(w^+) = w^+$ for $w_* < w^+ < \hat{w}$, if $\hat{w} \leq w^*$. To see this note that if $\hat{w} = w^*$ then $\lim_{w^+ \uparrow \hat{w}} w(w^+) > \hat{w}$ by assumption and if $\hat{w} < w^*$ then $w(\hat{w}) = \hat{w} + s(\hat{w})$ by (32) and (33). In the latter case if $w(\hat{w}) = \hat{w}$ then $Y(w^+) = u(w^+)$ and $e(w^+) = 0$ for $w^+ > \hat{w}$. But this violate $\hat{w} < w^*$. Were there w^+ such that $w(w^+) = w^+$ and $w_* < w^+ < \hat{w}$, let $\bar{\bar{w}} = \sup\{w^+ < \hat{w}: w(w^+) = w^+\}$. Clearly $\bar{\bar{w}} < \hat{w}$. Furthermore, $\dot{w}(w^+) \leq 0$ and $w(w^+) > w^+$ for $\bar{\bar{w}} < w^+ < \hat{w}$. Since w is continuous on $(w_*, \hat{w})$ by Lemma 2, $w(\bar{\bar{w}}) > \bar{\bar{w}}$. Hence $u(w^+) > w^+$ for $w_* < w^+ < \hat{w}$.

Since $\lambda_y(w^+) < 0$ for $w_* < w^+ < w^*$, a repetition of the above argument demonstrates that $w(w^+) > w^+$ and $\dot{w}(w^+) \leq 0$ for $w_* < w^+ < w^*$. Since (38) holds $\dot{s}(w^+) < 0$ as well over this interval.

Clearly $Y(w^*) > u(w^*)$. Since $\lambda_y(w^*) < 0$, it follows that $w^* < \overline{w}$. If $w^* = \overline{w}$, the transversality condition (34) could not be satisfied.

On $[w^*, \overline{w}]$ we must show that $e(w^+) = 0$. To do so we argue that λ_y cannot change signs on $[w^*, \overline{w}]$. On the contrary, if there were a w^+ such that $\lambda_y(w^+) = 0$ and $w^* < w^+ < \overline{w}$, then $\lambda_y(\tilde{w}) > 0$ for $\tilde{w} > w^+$, by the proof of Lemma 1. Then by either invoking the transversality condition (34) or an income smoothing argument,[11] one can show that the contract is nonoptimal.

Since $e \geq 0$ is required, it is necessary that $\dot{e}(w^+) \geq 0$ when $e(w^+) = 0$. Since $\lambda_y(w^+) < 0$ and (38) is satisfied for $w^* < w^+ < \overline{w}$, $\dot{e}(w^+) > 0$ is not possible over this interval.

VI. Quits

From the previous section it is evident that when $\overline{U}(k) < U^{BE}$ all workers with opportunity wage greater than k are fully laid off with zero severance pay. Let $\hat{w}$ be implicitly defined by

$$(39) \qquad u(\hat{w})\int_{\underline{w}}^{\hat{w}} f(w^+|k)dw^+ + \int_{\hat{w}}^{\overline{w}} u(w^+)f(w^+|k)dw^+ = \overline{U}(k)$$

Theorem 4: Suppose $\overline{U}(k) < U^{BE}$ and u exhibits nondecreasing absolute risk aversion. Suppose in addition that

$$(*) \qquad \frac{F(w^+|k) \cdot f(\hat{w}|k) \cdot u'(w^+)}{F(\hat{w}|k) \cdot f(w^+|k) \cdot u'(\hat{w})} \leq 1, \text{ for } w^+ \leq \hat{w},$$

while $f(w^+|k)$ is nonincreasing for $w^+ > \hat{w}$. Then the optimal contract is given by

$$e(w^+) = 1 \text{ and } w(w^+) = \hat{w} \text{ for } w^+ < \hat{w},$$

$$e(w^+) = 0 \text{ and } s(w^+) = 0 \text{ for } w^+ > \hat{w}.$$

Proof: It is not hard to show that $k - w + s \leq 0$ if (37) holds and u exhibits nondecreasing absolute risk aversion. It follows that for $w^+ < k$, then $s(w^+) = 0$ when $e(w^+) < 1$.

If the contract specified in the theorem is optimal then $\lambda_u = \frac{1}{u'(\hat{w})}[1 - (k-\hat{w})\frac{f(\hat{w}|k)}{F(\hat{w}|k)}]$ and $\mu = \frac{f(w^+|k)}{u'(\hat{w})}$ for $w^+ \leq \hat{w}$. From (30) one then gets $\lambda_y = \frac{F(w^+|k)(k-\hat{w})f(\hat{w}|k)}{F(\hat{w}|k)u'(\hat{w})}$, for $w^+ \leq \hat{w}$. The condition (*) is sufficient for (31) to be satisfied with e = 1, for $w^+ \leq \hat{w}$. For (31) to be satisfied with e = 0, it is necessary that $\frac{(k-w^+)f(w^+|k)}{u'(w^+)} \leq \lambda_y$. But the left hand side of this last inequality is decreasing for $w^+ < k$ as long as $f(w^+|k)$ is nonincreasing.

When (*) of Theorem 4 is not satisfied or when the value of λ_y specified in the proof of Theorem 4 does not satisfy $\frac{(k-w^+)f(w^+|k)}{u'(w^+)} \leq \lambda_y$, for $w^+ > \hat{w}$, then the optimal contract will involve an interval where $0 < e < 1$ and $w > w^+$. In this case the contract is monotonic if $\dot{w} \geq 0$ over this interval.

VII. Conclusion

The optimal $\overline{U}(k)$ is still determined by (16), even under asymmetric information. It follows that $\lambda_u = \frac{1}{u'(w_0)}$ for all k. This implies that the full employment wage, $w(\underline{w})$, is state dependent! It is lower under layoffs than under quits. The optimal contract not only treats workers with different opportunities differently for given k, but also treats workers with identical opportunities differently for varied k.

Competitive equilibrium need not be Pareto optimal when the information asymmetries described in the paper prevail. The less risky the distribution over opportunity wages is the "closer" the optimal labor contract approximates the symmetric information optimum. There is a tradeoff between better smoothing income ex ante and better employing workers at their most efficient place of employment ex post. As a result, it is conceivable that a decrease in worker mobility might actually improve welfare. This suggests that an equilibrium analysis in the spirit of this paper would prove fruitful.

Footnotes

1. This idea is the basis of the original papers on implicit contracts by Azariadas, Baily, and Gordon. The intertemporal insurance aspect of the implicit contract has been stressed by Holmstrom (1983).

2. Asymmetrically, it is assumed that the contract perfectly binds the firm, e.g., in the second period the firm never makes layoffs other than those specified in the contract conditions. In effect, it is assumed that reputational concerns are sufficiently strong to deter the firm from breach.

3. In a longer horizon model one could allow for new entrants into the labor force in the second period. However, there is no reason for doing so in this paper.

4. This assumption allows me to abstract from all considerations of firm moral hazard associated with asymmetric information over k. For a survey of the literature on this problem see Hart (1983).

5. See Harris and Townsend (1980) and Myerson (1979) for a discussion of the revelation principle.

6. A contract which satisfies (25) may satisfy (24) without satisfying (26). However, the problem becomes much more complicated in this case. There is a second order necessary condition associated with (25). This condition is

$$-\dot{e}(w^+)u'(w^+ + s(w^+)) + \dot{s}(w^+)(1-e(w^+))u''(w^+ + s(w^+)) \geq 0.$$

This condition is found by taking the second derivative of the right hand side of (24) with respect to $\tilde{w}$, evaluated at $\tilde{w} = w^+$, and subtracting the result from the total derivative of (25) with respect to w^+. One could impose this second order condition as an additional constraint. (Note that $\dot{e}$ and $\dot{s}$ would have to be treated as controls in this case.) This condition is weaker than requiring monotonic contracts. But it is not sufficient for (24).

7. Under the alternate contract we have

$$E\pi = \int_{\underline{w}}^{k} f(w^+|k)dw^+[k-w^a(\underline{w})] +$$

$$\int_{k}^{\overline{w}} [e^a(k-w^a) + (1-e^a)(-s^a)]f(w^+|k)dw^+$$

From the way the alternate contract variables are defined

$$\frac{d[E\pi]}{d\delta} = \frac{1}{u'(w^a(\underline{w}))} \int_{k}^{\overline{w}} \frac{dY^*(w^+)}{d\delta} f(w^+|k)\ dw^+$$

$$- \int_{k}^{\overline{w}} \frac{1}{u'(w^a(w^+))} \frac{dY^*(w^+)}{d\delta} f(w^+|k)dw^+.$$

Note that $\frac{dY^*(w^+)}{d\delta} \leq 0$ and that $w^a(\underline{w}) < w^a(w^+)$ for $w^+ \geq k$ when $\delta < \delta^*$.

8. Suppose to the contrary that for every $\varepsilon > 0$, $e(w^+) < 1$ for $k < w^+ < k + \varepsilon$. Then an alternate contract can be constructed by first fully employing all workers whose opportunity wage is less than $k + \varepsilon$ at a wage equal to

$$u^{-1}\left(\int_{\underline{w}}^{k+\varepsilon} Y(w^+)f(w^+|k)dw^+ / \int_{\underline{w}}^{k+\varepsilon} f(w^+|k)dw^+\right)$$

while keeping the contract unaltered for $w^+ \geq k + \varepsilon$. To this new contract a second alteration can be made via income smoothing, to restore feasibility, as is done in the text. The final contract can then be viewed as a function of ε, as can the expected profit which results from the final contract. The effect on expected profit from an increase in ε is twofold. First there is greater employment. Second there is more income smoothing. For ε small the first effect is negligible since the hired workers have an opportunity wage almost equal to k. Therefore, the second effect dominates when ε is small. Consequently, the original contract could not have been optimal.

9. This condition is not necessary, i.e., it is possible that $\dot{e}$, $\dot{s} \leq 0$ and $\dot{w} > 0$.

10. The subsequent argument can be extended to allow for densities which satisfy $f'(w^+|k) \geq 0$ and $\frac{f'(w^+|k)}{f(w^+|k)} \leq \frac{f''(w^+|k)}{f'(w^+|k)}$. Further generalizations require a different argument.

11. The income smoothing argument is based on the following. First, Lemma 2 can be extended to allow for $\lambda_y > 0$. Though $-sf(w^+|k) + \lambda_y u'(w^+ + s) + \mu u(w^+ + s)$ need not be concave in s in this case, it can nevertheless be shown that any extremum is a maximum as long as u exhibits nondecreasing absolute risk aversion. It follows that the contract is continuous when $\lambda_y > 0$ as long as $Y(w^+) > u(w^+)$. Then

if $e(w^+) > 0$ and $u(w) > w^+ + s(w^+)$, income smoothing can be achieved by raising s over a small interval containing w^+ such that $Y^a(w^+) = Y(w^+)$. The complete argument is quite similar to the one given in the proof of Lemma 1 and is therefore not repeated.

References

Akerlof, George A. and Hajime Miyazaki, "The Implicit Contract Theory of Unemployment meets the Wage Bill Argument," Review of Economic Studies, 47 (1980), 321-338.

Arnott, Richard, Arthur Hosios, and Joseph Stiglitz, "Implicit Contracts, Labor Mobility and Unemployment," Queen's University Discussion Paper #543, 1983.

Azariados, Costas, "Implicit Contracts and Underemployment Equilibria," Journal of Political Economy, 83 (1975), 1183-1202.

Baily, Martin Neil, "Wages and Employment under Uncertain Demand," Review of Economic Studies, 41 (1974), 37-50.

Geanakoplos, John and Takatoshi Ito, "On Implicit Contracts with Costly Search and Involuntary Unemployment: General Equilibrium," (1984).

Gordon, Donald F., "A Neoclassicl Theory of Keynesian Unemployment," Economic Inquiry, 12 (1974), 431-459.

Harris, Milton and Robert M. Townsend, "Resource Allocation under Asymmetric Information," Econometrica, 49 (1981), 33-64.

Hart, Oliver, "Optimal Labor Contracts under Asymmetric Information: An Introduction," Review of Economic Studies, 50 (1983), 3-36.

Holmstrom, Bengt, "Equilibrium Long-Term Labor Contracts," Quarterly Journal of Economics (Supplement), 98 (1983), 23-54.

Kahn, Charles M., "Optimal Severance Pay with Incomplete Information," Journal of Political Economy, 93 (1985), 435-456.

Myerson, Roger, "Incentive Compatibility and the Bargaining Problem," Econometrica, 47 (1979), 61-74.